

NJCCCS AREA: Mathematics
North Brunswick Township Public Schools

AP CALCULUS AB

Acknowledgements

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Diane M. Galella, Supervisor of Mathematics

Date: New_____

Revision May 2012

Board Adoption_____

This curriculum is aligned with the Advanced Placement Calculus AB syllabus submitted to and approved by the College Boards.

New Jersey Core Curriculum Content Standard Area: Mathematics

Topic/Course: AP Calculus AB

Grade: 12

Date: May 2012

Essential Question NJCCC Standard	NJCCCS – Skills/Objectives	Instructional Strategies Activities/ Materials / Technology Interdisciplinary Connections Cultural Diversity	Modifications ESL Special Education Academic Support/G&T	Assessments Formative Summative Benchmarks	PACING
<p>4.3.B Patterns and Algebra</p> <p>How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?</p>	<p>4.3.12 B2 (4.3.12 A2; 4.3.12 B4; 4.5 A2; 4.5 C2; 4.5 E2,3; 4.5 F3,5) Obj 2.1 Calculate average and instantaneous speed. Define and calculate limits for function values and apply the properties of limits. Use the Sandwich Theorem to find certain limits indirectly.</p> <p>Obj 2.2 Find and verify end behavior models for various functions. Calculate limits as $x \rightarrow \pm\infty$ and to identify vertical and horizontal asymptotes.</p> <p>Obj 2.3 Identify the interval upon which a given function is continuous and understand the meaning of a continuous function. Remove removable discontinuities by extending or modifying a function. Apply the Intermediate Value Theorem and the properties of algebraic combinations and composites of continuous functions.</p> <p>Obj 2.4 Apply directly the definition of the slope of a curve in order to calculate slope. Find the equations of the tangent line and normal line to a curve at a given point. Find the average rate of change of a function.</p>	<p>Activities:</p> <ul style="list-style-type: none"> The teacher and students will evaluate limits using various methods including graphical, analytical and numerical. Explore properties of limits of two functions by evaluating each one separately as $x \rightarrow c$ and comparing it to the limit of their product as $x \rightarrow c$ (textbook page 77 #65.) Explore the use of factoring to remove a discontinuity (textbook page 81.) Concepts worksheet 2.2-2.3 Evaluating limits using end-behavior models and evaluating continuity at $x = c$. <p>Materials: <u>Calculus: Graphical, Numerical, Algebraic</u> Finney/Demana/Waits/Kennedy; Pearson Education, Inc., 2007. <u>Teaching AP Calculus</u>, Lin McMullin, D & S Marketing Systems, Inc, 2005. <u>Calculus</u>, Foerster, Paul A., Key Curriculum Press, 1998.</p> <p>Technology: <i>Calculus In Motion</i>TM software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com</p>	<ul style="list-style-type: none"> Graphing calculator 	<p>Formative</p> <ul style="list-style-type: none"> In class practice problems Concepts worksheets Practice AP Questions <p>Summative</p> <ul style="list-style-type: none"> Precalculus (Review) Test Chapter 1, 2 Test 	22 days

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<p>4.3.B Patterns and Algebra</p> <p>How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?</p> <p>(CONTINUED)</p>	<p>4.3.12 B2 (4.3.12 A2; 4.3.12 B4; 4.5 A2; 4.5 C2; 4.5 E2,3; 4.5 F1,3,5)</p> <p>Obj 3.1 Calculate slopes and derivatives using the definition of a derivative. Graph a function from the graph of a derivative and graph a derivative from the graph of a function. Graph the derivative of a function given numerically with data.</p> <p>Obj 3.2 Find where a function is not differentiable and distinguish between corners, cusps, discontinuities, vertical and horizontal tangents. Analyze differentiable functions using a graphing calculator to explore local linearity. Approximate derivatives numerically and graphically.</p> <p>Obj 3.3 Use the rules of differentiation to calculate derivatives, including second and higher order derivatives. Use the derivative to calculate the instantaneous rate of change.</p> <p>Obj 3.4 Use derivatives to analyze straight-line motion and solve other problems involving rates of change.</p> <p>Obj 3.5 Use the rules of differentiating the six basic trigonometric functions.</p>	<p>Activities:</p> <ul style="list-style-type: none"> • Exploration 1-1 from Foerster: Students explore concept of a derivative with door-closer example • Explore local linearity using a graphing calculator to “zoom in” to see if the following functions are differentiable at $x = 0$. <ul style="list-style-type: none"> a) $f(x) = x + 1$ b) $g(x) = \sqrt{x^2 + 0.0001} + 0.99$ (textbook page 110) • Group activity: Students will be given two sets of cards containing graphs of functions on one set and graphs of their derivatives on the other. In small groups, students will use their knowledge of functions and derivatives to match a function’s graph with the graph of its derivative. • Concepts worksheet 3.1-3.3: Students will be able to make conclusions about the differentiability of functions based on information taken from tables and graphs. • Concepts worksheet 3.4: Students will work together to describe a particle’s motion using position, velocity and acceleration graphs. Students will also be able to make graphs describing a particle’s motion using position, velocity and acceleration functions. 	<ul style="list-style-type: none"> • Graphing calculator 	<p>Formative</p> <ul style="list-style-type: none"> • In class practice problems • Concepts worksheets • Practice AP Questions <p>Summative</p> <ul style="list-style-type: none"> • Quiz 3.1-3.3 • Quiz 3.4-3.6 • Chapter 3 Test 	31 days

	<p>Obj 3.6 Differentiate composite functions using the Chain Rule. Find slopes of parametrized curves.</p> <p>Obj 3.7 Find derivatives using implicit differentiation. Find the derivatives using the Power Rule for rational powers of x.</p> <p>Obj 3.8 Calculate derivatives of functions involving the inverse trigonometric functions.</p> <p>Obj 3.9 Calculate derivatives of exponential and logarithmic functions.</p>	<ul style="list-style-type: none"> • Explore the motion of a particle moving along a horizontal line $y = 2$ whose position is given by $x(t) = 4t^3 - 16t^2 + 15t$ for $t \geq 0$ (textbook page 132.) • Explore finding a derivative on an inverse graph geometrically when $f(x) = x^5 + 2x - 1$. (textbook page 166) • Concepts worksheet 3.8: Students will be able to identify inverse functions and use their knowledge to draw conclusions about the inverse function and its derivative. • Explore the rate at which milk warms when taken from a refrigerator and left on the counter on a warm summer day. (textbook page 174) <p>Materials: <u>Calculus: Graphical, Numerical, Algebraic</u> Finney/Demana/Waits/Kennedy; Pearson Education, Inc., 2007. <u>Teaching AP Calculus</u>, Lin McMullin, D & S Marketing Systems, Inc, 2005. <u>Calculus</u>, Foerster, Paul A., Key Curriculum Press, 1998.</p> <ul style="list-style-type: none"> • <p>Technology: <i>Calculus In Motion</i>TM software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com</p>			
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<p>4.3.B Patterns and Algebra</p> <p>How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?</p> <p>(CONTINUED)</p>	<p>4.3.12 B2 (4.3.12 A2; 4.3.12 B4; 4.5 A2; 4.5 C2; 4.5 E2,3; 4.5 F1,3,4)</p> <p>Obj 4.1 Determine the local or global extreme values of a function.</p> <p>Obj 4.2 Apply the Mean Value Theorem and find the intervals on which a function is increasing or decreasing.</p> <p>Obj 4.3 Use the First and Second Derivative Tests to determine the local extreme values of a function. Determine the concavity of a function and locate the points of inflection by analyzing the second derivative. Graph a function using information about its derivative.</p> <p>Obj 4.4 Solve application problems involving minimum or maximum values of a function.</p> <p>Obj 4.5 Find a linearization of a function. Estimate the change in a function using differentials.</p> <p>Obj 4.6 Solve related rate problems.</p>	<p>Activities:</p> <ul style="list-style-type: none"> The teacher and students will explore extreme values and the derivative of a function at its extreme values using the graphing calculator. (textbook p.193) Concepts worksheet 4.1: Students will use derivatives to identify extreme values and points of inflection, as well as intervals on which a function is increasing, decreasing, concave up or concave down. Explore local extrema by analyzing derivative data from a table. (textbook p.204 #57) Concepts worksheet 4.2: Students will use graphs and tables to demonstrate their knowledge of the Mean Value Theorem. Concept worksheet 4.3: Students will use information about first and second derivatives to sketch the graph of a function. Group Activity: Students will work together to minimize fuel consumption and optimize fuel efficiency of an automobile given $C(v)$, a function for the rate of fuel consumption with respect to velocity. Group Project: Students will be given materials and instructed to create an open rectangular box with the largest possible volume. Students will submit all calculations to support their model. 	<ul style="list-style-type: none"> Graphing calculator 	<p>Formative</p> <ul style="list-style-type: none"> In class practice problems Concepts worksheets Practice AP Questions <p>Summative</p> <ul style="list-style-type: none"> Group Project Quiz 4.1-4.3 Chapter 4 Test 	25 days

		<p>Materials: <u>Calculus: Graphical, Numerical, Algebraic</u> Finney/Demana/Waits/Kennedy; Pearson Education, Inc., 2007. <u>Teaching AP Calculus</u>, Lin McMullin, D & S Marketing Systems, Inc, 2005. <u>Calculus</u>, Foerster, Paul A., Key Curriculum Press, 1998.</p> <p>•</p> <p>Technology: <i>Calculus In Motion</i>[™] software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com</p>			
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<p>4.3.B Patterns and Algebra</p> <p>How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?</p> <p>(CONTINUED)</p>	<p>4.3.12 B2 (4.3.12 A2; 4.3.12 B4; 4.5 A2; 4.5 C2; 4.5 E2,3; 4.5 F1)</p> <p>Obj 5.1 Approximate the area under the graph of a nonnegative continuous function by using the rectangular approximation methods. Interpret the area under a graph as a net accumulation of a rate of change.</p> <p>Obj 5.2 Express the area under a curve as a definite integral and as a limit of Reimann sums. Compute the area under a curve using a numerical integration procedure.</p> <p>Obj 5.3 Apply the rules for definite integrals and find the average value of a function over a closed interval.</p> <p>Obj 5.4 Apply the Fundamental Theorem of Calculus. Understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.</p> <p>Obj 5.5 Approximate the definite integral by using the Trapezoidal Rule.</p>	<p>Activities:</p> <ul style="list-style-type: none"> Explore the area of a circle by computing the limit of the area of an inscribed regular polygon as $n \rightarrow \infty$. Explain how this computation parallels the computation used in the rectangular approximation method for estimating the area under a curve. (textbook p.273 #37 and 40) Explore the use of the graphing calculator to graph the function $F(x) = \int_3^x \tan t \, dt + 5.$ (textbook p.298) Explore the effect of changing a in $\int_a^x f(t) \, dt.$ (textbook p.299) Concepts worksheet 5.3-5.4: Students will sketch the graph of a function given the graph of its derivative. Concepts worksheet 5.4: Students will test their understanding of Parts 1 and 2 of The Fundamental Theorem of Calculus. Group Activity: (group activity exploration 5.4) Students will derive the formula for the volume of a right circular cone using a Riemann Sum of the volumes of cylindrical disks. <p>Materials: Calculus: Graphical, Numerical,</p>	<ul style="list-style-type: none"> Graphing calculator 	<p>Formative</p> <ul style="list-style-type: none"> In class practice problems Concepts worksheets Practice AP Questions <p>Summative</p> <ul style="list-style-type: none"> Quiz 5.1-5.3 Chapter 5 Test 	26 days

		<p><u>Algebraic</u> Finney/Demana/Waits/ Kennedy; Pearson Education, Inc., 2007.</p> <p><u>Teaching AP Calculus</u>, Lin McMullin, D & S Marketing Systems, Inc, 2005.</p> <p><u>Calculus</u>, Foerster, Paul A., Key Curriculum Press, 1998.</p> <p>•</p> <p>Technology: <i>Calculus In Motion</i>[™] software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com</p>			
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<p>4.3.B Patterns and Algebra</p> <p>How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?</p>	<p>4.3.12 B2 (4.3.12 A2; 4.3.12 B4; 4.3.12 C1; 4.5 A2; 4.5 C2; 4.5 D3; 4.5 E2,3; 4.5 F1)</p> <p>Obj 6.1 Construct antiderivatives using the Fundamental Theorem of Calculus. Solve initial value problems of the form $\frac{dy}{dx} = f(x)$, $y_0 = f(x_0)$.</p> <p>Construct slope fields using technology and interpret slope fields as visualizations of differential equations.</p> <p>Obj 6.2 Compute definite and indefinite integrals by the method of substitution.</p> <p>Obj 6.4 Solve problems involving exponential growth and decay in a variety of applications.</p>	<p>Activities:</p> <ul style="list-style-type: none"> Group activity: Students will create a slope field for a given differential equation and plot a possible solution to the equation based on a given initial value. Students will then solve the differential equation analytically and graph it on a graphing calculator to confirm the sketch on the slope field. Explore different approaches to the substitution method for evaluating a definite integral. (textbook p.339 #67 and # 68) Show your knowledge of The Law of Exponential Change by making a table identifying the differential equation and initial values for each of the formulas used in the exponential growth and decay problems. <p>Materials: <u>Calculus: Graphical, Numerical, Algebraic</u> Finney/Demana/Waits/Kennedy; Pearson Education, Inc., 2007. <u>Teaching AP Calculus</u>, Lin McMullin, D & S Marketing Systems, Inc, 2005. <u>Calculus</u>, Foerster, Paul A., Key Curriculum Press, 1998.</p> <p>•</p> <p>Technology: <i>Calculus In Motion</i>™ software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com</p>	<ul style="list-style-type: none"> Graphing calculator 	<p>Formative</p> <ul style="list-style-type: none"> In class practice problems Concepts worksheets Practice AP Questions <p>Summative</p> <ul style="list-style-type: none"> Quiz 6.1-6.2 Chapter 6 Test 	<p>20 days</p>

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<p>4.3.B Patterns and Algebra</p> <p>How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?</p>	<p>4.3.12 B2 (4.3.12 A2; 4.3.12 B4; 4.3.12 C1; 4.5 A2; 4.5 C2; 4.5 D3; 4.5 E2,3; 4.5 F1)</p> <p>Obj 7.1 Solve problems in which a rate is integrated to find the net change over time in a variety of applications.</p> <p>Obj 7.2 Use integration to calculate areas of regions in a plane.</p> <p>Obj 7.3 Use integration (by the slice method) to calculate volumes of solids.</p> <p><i>AP Exam Review & Exam</i></p> <p>Obj 7.4 Use integration to calculate surface areas of solids of revolution.</p> <p>Obj 7.5 Use integration to calculate lengths of curves in a plane.</p> <p>Obj 7.6 Find limits of indeterminate forms using L'Hopital's Rule.</p>	<p>Activities:</p> <ul style="list-style-type: none"> • Concepts worksheet 7.1-7.5: Students will use integrals to determine the net change in a quantity over a period of time. • Explore the area of an ellipse by finding the formulas for the semi-ellipses and integrating using a graphing calculator. (textbook p.398 #56) • Concepts worksheet 7.2: Students will demonstrate ability to determine area between two curves with respect to the x or y-axis. • Concepts worksheet 7.3: Students will demonstrate the ability to determine the volumes of solids of revolution. • Group Activity: Students will derive the formula for the volume of a "cone" with irregular polygonal base using a Riemann Sum of volumes of thin slabs with thickness Δx. <p>Materials: <u>Calculus: Graphical, Numerical</u>, Pearson Education, Inc., 2007. <u>Teaching AP Calculus</u>, Lin McMullin, D & S Marketing Systems, Inc, 2005. <u>Calculus</u>, Foerster, Paul A., Key Curriculum Press, 1998.</p> <p>Technology: <i>Calculus In Motion™</i> software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com</p>	<ul style="list-style-type: none"> • Graphing calculator 	<p>Formative</p> <ul style="list-style-type: none"> • In class practice problems • Concepts worksheets • Practice AP Questions <p>Summative</p> <ul style="list-style-type: none"> • Chapter 7 Test • AP Exam 	<p>21 days</p>

NORTH BRUNSWICK TOWNSHIP HIGH SCHOOL

(2425) AP Calculus AB

Grades 11, 12

5 Credits - 1 year

Course Description:

Advanced Placement Calculus AB is a study of calculus at the college level. Emphasis will be placed on the core curriculum provided by the College Entrance Examination Board (CEEB). It is expected that each student will take the AB level of the Advanced Placement Calculus Exam. A TI-83+ or better graphing calculator is necessary for success. Students must complete a summer assignment.

Students should be able to

- work with functions represented in a variety of ways: graphical, numerical, analytical, or vertical. They should understand the connections among these representations;
- understand the meaning of the derivative in terms of a rate of change and local linear approximation and they should be able to use derivatives to solve a variety of problems;
- understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of a rate of change and should be able to use integrals to solve a variety of problems;
- understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus;
- communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems;
- model a written description of a physical situation with a function, a differential equation, or an integral;
- use technology to help solve problems, experiment, interpret results, and verify conclusions;
- determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement;
- develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

The following is taken from the College Boards Acorn Book listing topics to be tested on the Advanced Placement Calculus AB/BC exams.

I. Functions, Graphs and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process.
- Calculating limits using algebra.

- Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior.
- Describing asymptotic behavior in terms of limits involving infinity.
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth and logarithmic growth).

Continuity as a property of functions

- An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits.
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

Parametric, polar and vector functions. The analysis of planar curves includes those given in parametric form, polar form and vector form.

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically and analytically.
- Derivative interpreted as an instantaneous rate of change.
- Derivative defined as the limit of the difference quotient.
- Relationship between differentiability and continuity.

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation.
- Instantaneous rate of change as the limit of average rate of change.
- Approximate rate of change from graphs and tables of values.

Derivative as a function

- Corresponding characteristics of graphs of f and f' .
- Relationship between the increasing and decreasing behavior of f and the sign of f' .
- The Mean Value Theorem and its geometric interpretation.
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f , f' and f'' .
- Relationship between the concavity of f and the sign of f'' .
- Points of inflection as places where concavity changes.

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity.

- Optimization, both absolute (global) and relative (local) extrema.
- Modeling rates of change, including related rates problems.
- Use of implicit differentiation to find the derivative of an inverse function.
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed and acceleration.
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric and inverse trigonometric functions.
- Derivative rules for sums, products and quotients of functions.
- Chain rule and implicit differentiation.

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums.
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f'(x)dx = f(b) - f(a)$
- Basic properties of definite integrals (examples include additivity and linearity).

Applications of integrals. Appropriate integrals are used in a variety of applications to model physical, biological or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals.
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions.
- Antiderivatives by substitution of variables (including change of limits for definite integrals).

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line.
- Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth).

Numerical approximations to definite integrals. Use of Riemann sums (using left, right and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically and by tables of values.

Evaluation:

Marking Period grades will be determined as follows:

Performance assessments	90%
Homework	10%

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