NJCCCS AREA: Mathematics

North Brunswick Township Public Schools

AP CALCULUS BC

Acknowledgements

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Date: New_____________________

Revision May 2012

Board Adoption_____________

This curriculum is aligned with the Advanced Placement Calculus BC syllabus submitted to and approved by the College Boards.

New Jersey Core Curriculum Content Standard Area: Mathematics

Topic/Course: AP Calculus BC Grade: 12 Date: May 2012

Essential Question NJCCC Standard	**NJCCCS –** Skills/Objectives	**Instructional Strategies** Activities/ Materials / Technology Interdisciplinary Connections Cultural Diversity	**Modifications** ESL Special Education Academic Support/G&T	**Assessments** Formative Summative Benchmarks	**PACING**
4.3.B Patterns and Algebra How are patterns of change related to the behavior of functions?	**4.3.12 B2 (4.3.12 A2; 4.3.12 B4; 4.5 A2; 4.5 C2; 4.5 E2,3; 4.5 F1,3,5)** **Obj 2.1** Calculate average and instantaneous speed. Define and calculate limits for function values and apply the properties of limits. Use the Sandwich Theorem to find certain limits indirectly. **Obj 2.2** Find and verify end behavior models for various functions. Calculate limits as $x \to \pm\infty$ $x \to \pm\infty$ and to identify vertical and horizontal asymptotes. **Obj 2.3** Identify the interval upon which a given function is continuous and understand the meaning of a continuous function. Remove removable discontinuities by extending or modifying a function. Apply the Intermediate Value Theorem and the properties of algebraic combinations and composites of continuous functions. **Obj 2.4** Apply directly the definition of the slope of a curve in order to calculate slope. Find the equations of the tangent line and normal line to a curve at a given point. Find the average rate of change of a function.	**Activities:** • The teacher and students will evaluate limits using various methods including graphical, analytical and numerical. • Explore properties of limits of two functions by evaluating each one separately as $x \to c$ and comparing it to the limit of their product as $x \to c$ (textbook page 77 #65). • Explore the use of factoring to remove a discontinuity (textbook page 81). • Concepts worksheet 2.2-2.3 Evaluating limits using end-behavior models and evaluating continuity at $x = c$. **Materials:** Calculus: Graphical, Numerical, Algebraic Finney/Demana/Waits/ Kennedy; Pearson Education, Inc., 2007. Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005. Calculus, Foerster, Paul A., Key Curriculum Press, 1998. **Technology:** *Calculus In Motion*™ software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com	• Graphing calculator	**Formative** • In class practice problems • Concepts worksheets • Practice AP Questions **Summative** • Precaluclus (Review) Test • Chapter 1, 2 Test	7 days

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4.3.B Patterns and Algebra How are patterns of change related to the behavior of functions? **(continued)**	**4.3.12 B2 (4.3.12 A2; 4.3.12 B4; 4.5 A2; 4.5 C2; 4.5 E2,3; 4.5 F1,3,5)** **<u>Obj 3.1</u>** Calculate slopes and derivatives using the definition of a derivative. Graph a function from the graph of a derivative and graph a derivative from the graph of a function. Graph the derivative of a function given numerically with data. **<u>Obj 3.2</u>** Find where a function is not differentiable and distinguish between corners, cusps, discontinuities, vertical and horizontal tangents. Analyze differentiable functions using a graphing calculator to explore local linearity. Approximate derivatives numerically and graphically. **<u>Obj 3.3</u>** Use the rules of differentiation to calculate derivatives, including second and higher order derivatives. Ue the derivative to calculate the instantaneous rate of change. **<u>Obj 3.4</u>** Use derivatives to analyze straight-line motion and solve other problems involving rates of change. **<u>Obj 3.5</u>** Use the rules of differentiating the six basic trigonometric functions.	**<u>Activities:</u>** • Exploration 1-1 from Foerster: Students explore concept of a derivative with door-closer example. • Explore local linearity using a graphing calculator to “zoom in” to see if the following functions are differentiable at $x = 0$. a) $f(x) = \|x\| + 1$ b) $g(x) = \sqrt{x^2 + 0.0001} + 0.99$ (textbook page 110). • <u>Group activity</u>: Students will be given two sets of cards containing graphs of functions on one set and graphs of their derivatives on the other. In small groups, students will use their knowledge of functions and derivatives to match a function’s graph with the graph of its derivative. • Concepts worksheet 3.1-3.3: Students will be able to make conclusions about the differentiability of functions based on information taken from tables and graphs. • Concepts worksheet 3.4: Students will work together to describe a particle’s motion using position, velocity and acceleration graphs. Students will also be able to make graphs describing a particle’s motion using position, velocity and acceleration functions.	• Graphing calculator	**Formative** • In class practice problems • Concepts worksheets • Practice AP Questions **Summative** • Quiz 3.1-3.3 • Quiz 3.4-3.6 • Chapter 3 Test	23 days

	Obj 3.6 Differentiate composite functions using the Chain Rule. Find slopes of parametrized curves. **Obj 3.7** Find derivatives using implicit differentiation. Find the derivatives using the Power Rule for rational powers of *x*. **Obj 3.8** Calculate derivatives of functions involving the inverse trigonometric functions. **Obj 3.9** Calculate derivatives of exponential and logarithmic functions.	• Explore the motion of a particle moving along a horizontal line $y = 2$ whose position is given by $x(t) = 4t^3 - 16t^2 + 15t$ for $t \geq 0$ (textbook page 132). • Explore finding a derivative on an inverse graph geometrically when $f(x) = x^5 + 2x - 1$ (textbook page 166). • Concepts worksheet 3.8: Students will be able to identify inverse functions and use their knowledge to draw conclusions about the inverse function and its derivative. • Explore the rate at which milk warms when taken from a refrigerator and left on the counter on a warm summer day (textbook page 174). **Materials:** Calculus: Graphical, Numerical, Algebraic Finney/Demana/Waits/Kennedy; Pearson Education, Inc., 2007. Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005. Calculus, Foerster, Paul A., Key Curriculum Press, 1998. **Technology:** *Calculus In Motion*™ software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com			

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4.3.B Patterns and Algebra How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations? (continued)	**4.3.12 B2 (4.3.12 A2; 4.3.12 B4; 4.5 A2; 4.5 C2; 4.5 E2,3; 4.5 F1,3,4)** **Obj 4.1** Determine the local or global extreme values of a function. **Obj 4.2** Apply the Mean Value Theorem and find the intervals on which a function is increasing or decreasing. **Obj 4.3** Use the First and Second Derivative Tests to determine the local extreme values of a function. Determine the concavity of a function and locate the points of inflection by analyzing the second derivative. Graph a function using information about its derivative. **Obj 4.4** Solve application problems involving minimum or maximum values of a function. **Obj 4.5** Find a linearization of a function. Estimate the change in a function using differentials. **Obj 4.6** Solve related rate problems.	**Activities:** • The teacher and students will explore extreme values and the derivative of a function at its extreme values using the graphing calculator (textbook p.193). • Concepts worksheet 4.1: Students will use derivatives to identify extreme values and points of inflection, as well as intervals on which a function is increasing, decreasing, concave up or concave down. • Explore local extrema by analyzing derivative data from a table (textbook p.204 #57). • Concepts worksheet 4.2: Students will use graphs and tables to demonstrate their knowledge of the Mean Value Theorem. • Concept worksheet 4.3: Students will use information about first and second derivatives to sketch the graph of a function. • Group Activity: Students will work together to minimize fuel consumption and optimize fuel efficiency of an automobile given *C(v)*, a function for the rate of fuel consumption with respect to velocity.	• Graphing calculator	**Formative** • In class practice problems • Concepts worksheets • Practice AP Questions **Summative** • Group Project • Quiz 4.1-4.3 • Chapter 4 Test	15 days

		• Group Project: Students will be given materials and instructed to create an open rectangular box with the largest possible volume. Students will submit all calculations to support their model. • Explore Newton's Method for approximating a solution to the equation $f(x)=0$ using a graphing calculator (textbook p.237). **Materials:** Calculus: Graphical, Numerical, Algebraic Finney/Demana/Waits/ Kennedy; Pearson Education, Inc., 2007. Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005. Calculus, Foerster, Paul A., Key Curriculum Press, 1998. **Technology:** *Calculus In Motion*™ software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com			

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4.3.B Patterns and Algebra How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations? (continued)	**4.3.12 B2 (4.3.12 A2; 4.3.12 B4; 4.5 A2; 4.5 C2; 4.5 E2,3; 4.5 F1)** **<u>Obj 5.1</u>** Approximate the area under the graph of a nonnegative continuous function by using the rectangular approximation methods. Interpret the area under a graph as a net accumulation of a rate of change. **<u>Obj 5.2</u>** Express the area under a curve as a definite integral and as a limit of Reimann sums. Compute the area under a curve using a numerical integration procedure. **<u>Obj 5.3</u>** Apply the rules for definite integrals and find the average value of a function over a closed interval. **<u>Obj 5.4</u>** Apply the Fundamental Theorem of Calculus. Understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus. **<u>Obj 5.5</u>** Approximate the definite integral by using the Trapezoidal Rule.	**<u>Activities:</u>** • Explore the area of a circle by computing the limit of the area of an inscribed regular polygon as $n \to \infty$. Explain how this computation parallels the computation used in the rectangular approximation method for estimating the area under a curve. (textbook p.273 #37 and 40 • Explore the use of the graphing calculator to graph the function $F(x) = \int_3^x \tan t \, dt + 5$. (textbook p.298 • Explore the effect of changing a in $\int_a^x f(t)\, dt$. (textbook p.299 • Concepts worksheet 5.3-5.4: Students will sketch the graph of a function given the graph of its derivative. • Concepts worksheet 5.4: Students will test their understanding of Parts 1 and 2 of The Fundamental Theorem of Calculus. • <u>Group Activity</u>: (group activity exploration 5.4) Students will derive the formula for the volume of a right circular cone using a Riemann Sum of the volumes of cylindrical disks.	• Graphing calculator	**Formative** • In class practice problems • Concepts worksheets • Practice AP Questions **Summative** • Quiz 5.1-5.3 • Chapter 5 Test	16 days

		Materials: Calculus: Graphical, Numerical, Algebraic Finney/Demana/Waits/ Kennedy; Pearson Education, Inc., 2007. Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005. Calculus, Foerster, Paul A., Key Curriculum Press, 1998. **Technology:** *Calculus In Motion*™ software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com			

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4.3.B Patterns and Algebra How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?	**4.3.12 B2 (4.3.12 A2; 4.3.12 B4; 4.3.12 C1; 4.5 A2; 4.5 C2; 4.5 D3; 4.5 E2,3; 4.5 F1)** **Obj 6.1** Construct antiderivatives using the Fundamental Theorem of Calculus. Solve initial value problems of the form $\frac{dy}{dx}=f(x),\ y_0=f(x_0)$.. Construct slope fields using technology and interpret slope fields as visualizations of differential equations. **Obj 6.2** Compute definite and indefinite integrals by the method of substitution. **Obj 6.3** Using integration by parts to evaluate indefinite and definite integrals. Use tabular integration to evaluate integrals that require repeated use of integration by parts. **Obj 6.4** Solve problems involving exponential growth and decay in a variety of applications.	**Activities:** • Group activity: Students will create a slope field for a given differential equation and plot a possible solution to the equation based on a given initial value. Students will then solve the differential equation analytically and graph it on a graphing calculator to confirm the sketch on the slope field. • Explore different approaches to the substitution method for evaluating a definite integral. (textbook p.339 #67 and # 68) • Show your knowledge of The Law of Exponential Change by making a table identifying the differential equation and initial values for each of the formulas used in the exponential growth and decay problems. **Materials:** Calculus: Graphical, Numerical, Algebraic Finney/Demana/Waits/ Kennedy; Pearson Education, Inc., 2007. Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005. Calculus, Foerster, Paul A., Key Curriculum Press, 1998. **Technology:** *Calculus In Motion*™ software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com	• Graphing calculator	**Formative** • In class practice problems • Concepts worksheets • Practice AP Questions **Summative** • Quiz 6.1-6.2 • Chapter 6 Test	20 days

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4.3.B Patterns and Algebra How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?	**4.3.12 B2 (4.3.12 A2; 4.3.12 B4; 4.3.12 C1; 4.5 A2; 4.5 C2; 4.5 D3; 4.5 E2,3; 4.5 F1)** **Obj 7.1** Solve problems in which a rate is integrated to find the net change over time in a variety of applications. **Obj 7.2** Use integration to calculate areas of regions in a plane. **Obj 7.3** Use integration (by slices or shells) to calculate volumes of solids. **Obj 7.4** Use integration to calculate lengths of curves in a plane. **Obj 7.5** Use integral calculus to model a variety of problems involving ratest of change in a variety of applications, possibly in unfamiliar contexts.	**Activities:** • Concepts worksheet 7.1-7.5: Students will use integrals to determine the net change in a quantity over a period of time. • Explore the area of an ellipse by finding the formulas for the semi-ellipses and integrating using a graphing calculator (textbook p.398 #56). • Concepts worksheet 7.2: Students will demonstrate ability to determine area between two curves with respect to the x or y-axis. • Concepts worksheet 7.3: Students will demonstrate the ability to determine the volumes of solids of revolution. • Group Activity: Students will derive the formula for the volume of a "cone" with irregular polygonal base using a Riemann Sum of volumes of thin slabs with thickness Δx. **Materials:** Calculus: Graphical, Numerical, Algebraic; Pearson Education, Inc., 2007. Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005. Calculus, Foerster, Paul A., Key Curriculum Press, 1998. **Technology:** *Calculus In Motion*™ software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com	• Graphing calculator	**Formative** • In class practice problems • Concepts worksheets • Practice AP Questions **Summative** • Quiz 7.1-7.3 • Chapter 7 Test	15 days

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4.3.A Patterns and Algebra How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?	**4.3.12A (4.3.12 B2; 4.3.12 C1; 4.5 A2; 4.5 C2; 4.5 D3; 4.5 E2,3; 4.5 F1)** **<u>Obj 8.1</u>** Define sequences explicitly or recursively. Define explicit and recursive rules for arithmetic and geometric sequences. Graph sequences using parametric or graphing mode. Use properties of limits to find the limit of a sequence. Determine whether a sequence converges or diverges and find its limit using the Sandwich Theorem for Sequences or the Absolute Value Theorem. **<u>Obj 8.2</u>** Find limits of indeterminate form using l'Hôpital's Rule. **<u>Obj 8.3</u>** Use l'Hôpital's Rule to compare the rates of growth of functions. **<u>Obj 8.4</u>** Use limits to evaluate improper integrals. Use the direct comparison test and the limit comparison test to determine the convergence or divergence of improper integrals.	**<u>Activities:</u>** • Explore l'Hôpital's Rule graphically for $f(x)=\frac{\sin x}{x}$ (textbook p.446). • Find examples of two functions *f(x)* and *g(x)* the limit of whose product evaluates to 0, a non-zero number, or ∞ (textbook p.452 #68,69). • Show that 3^x grows faster than 2^x which grows faster than x^2 as $x\to\infty$ (textbook p.454). • Explain why $\int_0^1 \frac{dx}{x^p}$ converges for 0<p<1 and diverges for p≥1 (textbook p.463). • Discover for which p-values $\int_1^\infty \frac{dx}{x^p}$ converges and for which it diverges (textbook p.468 #54). • <u>Group Activity</u>: Students will attempt to evaluate improper integrals on the graphing calculator. **<u>Materials:</u>** <u>Calculus: Graphical, Numerical, Algebraic</u>; Pearson Education, Inc., 2007. <u>Teaching AP Calculus</u>, Lin McMulllin, D & S Marketing Systems, Inc, 2005. **<u>Technology:</u>** *Calculus In Motion*™ software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: <u>www.winplot.com</u>	• Graphing calculator	**Formative** • In class practice problems • Concepts worksheets • Practice AP Questions **Summative** • Chapter 8 Test	8 days

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4.3.B Patterns and Algebra How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations? (continued)	**4.3.12 B2 (4.3.12 A; 4.3.12B; 4.3.12 C1; 4.5 A2; 4.5 C2; 4.5 D3; 4.5 E2,3; 4.5 F1)** **<u>Obj 9.1</u>** Apply the properties of geometric series. Differentiate, integrate, or substitute into a known power series to find additional power series representations. **<u>Obj 9.2</u>** Use derivatives to find the Maclaurin series or Taylor series generated by a differentiable function. **<u>Obj 9.3</u>** Approximate a function with a Taylor polynomial. Analyze the truncation error of a series using graphical methods or the Remainder Estimation Theorem. Use Euler's formula to relate the functions sin x, cos x, and e^x **<u>Obj 9.4</u>** Use the *n*th-Term Test, the Direct Comparison Test, and the Ratio Test to determine the convergence or divergence of a series of numbers or the radius of convergence of a power series. **<u>Obj 9.5</u>** Use the Integral Test and the Alternating Series Test to determine the convergence or divergence of a series of numbers. Determine the convergence or divergence of *p*-series, including the harmonic series. Determine the absolute convergence, conditional convergence, or divergence of a	**<u>Activities:</u>** • Given a power series for $\frac{1}{1-x}$, find power series for related functions (textbook p.477). • Given a power series for $\frac{1}{1+x^2}$, find the inverse tangent function (textbook p.480). • Differentiate the power series for e^x to see its unique properties. • Design a polynomial to specifications using derivatives (textbook p.484). • <u>Group Activity</u>: Construct a sixth-order Taylor polynomial and the Taylor series at *x*=0 for *cos x*, and observe the closer and closer approximations graphically. • Derive Euler's formula using the Maclaurin series for e^x, *cos x*, and *sin x*. (textbook p.499) • Use the Ratio Test and check endpoints to determine the radius of convergence for a series (textbook p.509). • Concepts worksheet 9.1-9.5: Power Series and Taylor's Theorem. • Use the Integral Test to derive the p-Series Test (textbook p.514). • Use a flowchart diagram (textbook p.521) to determine convergence for different types of series.	• Graphing calculator	**Formative** • In class practice problems • Concepts worksheets • Practice AP Questions **Summative** • 9.1 Quiz • 9.1 to 9.3 Quiz • Chapter 9 Test	23 days

	power series at the endpoints of its interval of convergence.	**Materials:** Calculus: Graphical, Numerical, Algebraic Finney/Demana/Waits/ Kennedy; Pearson Education, Inc., 2007. Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005. Calculus, Foerster, Paul A., Key Curriculum Press, 1998. **Technology:** *Calculus In Motion*™ software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com			

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4.3.B Patterns and Algebra How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?	**4.3.12 B2 (4.2.12C; 4.3.12 A; 4.3.12B; 4.3.12 C1; 4.5 A2; 4.5 C2; 4.5 D3; 4.5 E2,3; 4.5 F1)** **Obj 10.1** Find derivatives and second derivatives of parametrically defined functions. Calculate lengths of parametrically defined curves. **Obj 10.2** Represent vectors in the form <*a,b*> and perform algebraic computations involving vectors. Use vectors to solve problems involving the modeling of planar motion, velocity, acceleration, speed, and displacement and distance traveled. **Obj 10.3** Graph polar equations and determine the symmetry of polar graphs. Convert Cartesian equations into polar form and vice versa. Calculate slopes and areas of regions in the plane determined by polar curves.	**Activities:** • Explore a type of parametric function (cycloids) by graphing, finding its intercepts, and analyzing its derivatives (textbook p.534). • Concepts worksheet 10.2: Given a point on a vector, students will consider the possible initial points of a vector. Students will also find and analyze the first and second derivatives of vector-defined functions. • Concept worksheet 10.3: Students will consider families of polar curves. They will then write polar functions in parametric form and analyze the derivatives. **Materials:** Calculus: Graphical, Numerical, Algebraic Finney/Demana/Waits/ Kennedy; Pearson Education, Inc., 2007. Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005. Calculus, Foerster, Paul A., Key Curriculum Press, 1998. **Technology:** *Calculus In Motion*™ software, Audrey Weeks TI-84 plus graphing calculator Winplot graphing tool: www.winplot.com	• Graphing calculator	**Formative** • In class practice problems • Concepts worksheets • Practice AP Questions **Summative** • Chapter 10 Test	14 days

NORTH BRUNSWICK TOWNSHIP HIGH SCHOOL

(2505) AP Calculus BC
Grades 11, 12
5 Credits - 1 year

Course Description:
Advanced Placement Calculus BC is a study of calculus at the college level. Emphasis will be placed on the core curriculum provided by the College Entrance Examination Board (CEEB). It is expected that each student will take the BC level of the Advanced Placement Calculus Exam. A TI-83+ or better graphing calculator is necessary for success. Students must complete a summer assignment.

Students should be able to

- work with functions represented in a variety of ways: graphical, numerical, analytical, or vertical. They should understand the connections among theses representations;
- understand the meaning of the derivative in terms of a rate of change and local linear approximation and they should be able to use derivatives to solve a variety of problems;
- understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of a rate of change and should be able to use integrals to solve a variety of problems;
- understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus;
- communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems;
- model a written description of a physical situation with a function, a differential equation, or an integral;
- use technology to help solve problems, experiment, interpret results, and verify conclusions;
- determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement;
- develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

The following is taken from the College Boards Acorn Book listing topics to be tested on the Advanced Placement Calculus AB exams.

I. Functions, Graphs and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process.
- Calculating limits using algebra.
- Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior.•
- Describing asymptotic behavior in terms of limits involving infinity.
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth and logarithmic growth).

Continuity as a property of functions

- An intuitive understanding of continuity. (The function values can be made as
- close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits.
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically and analytically.
- Derivative interpreted as an instantaneous rate of change.
- Derivative defined as the limit of the difference quotient.
- Relationship between differentiability and continuity.

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation.
- Instantaneous rate of change as the limit of average rate of change.
- Approximate rate of change from graphs and tables of values.

Derivative as a function

- Corresponding characteristics of graphs of f and f'.
- Relationship between the increasing and decreasing behavior of f and the sign of f'.
- The Mean Value Theorem and its geometric interpretation.
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f, f' and f''.
- Relationship between the concavity of f and the sign of f''.
- Points of inflection as places where concavity changes.

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity.
- Optimization, both absolute (global) and relative (local) extrema.
- Modeling rates of change, including related rates problems.
- Use of implicit differentiation to find the derivative of an inverse function.
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed and acceleration.
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.
- Numerical solution of differential equations using Euler's method.
- L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series.

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric and inverse trigonometric functions.
- Derivative rules for sums, products and quotients of functions.
- Chain rule and implicit differentiation.
- Derivatives of parametric, polar and vector functions.

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums.
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f'(x)dx = f(b) - f(a)$
- Basic properties of definite integrals (examples include additivity and linearity).

Applications of integrals. Appropriate integrals are used in a variety of applications to model physical, biological or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.

Fundamental Theorem of Calculus
- Use of the Fundamental Theorem to evaluate definite integrals.
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Techniques of antidifferentiation
- Antiderivatives following directly from derivatives of basic functions.
- Antiderivatives by substitution of variables (including change of limits for definite integrals).
- Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only).
- Improper integrals (as limits of definite integrals).

Applications of antidifferentiation
- Finding specific antiderivatives using initial conditions, including applications to motion along a line.
- Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth).
- Solving logistic differential equations and using them in modeling.

Numerical approximations to definite integrals. Use of Riemann sums (using left, right and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically and by tables of values.

IV. Polynomial Approximations and Series

Concept of series. A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence and divergence.

Series of constants
- Motivating examples, including decimal expansion.
- Geometric series with applications.
- The harmonic series.
- Alternating series with error bound.
- Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series.
- The ratio test for convergence and divergence.
- Comparing series to test for convergence or divergence.

Taylor series
- Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve).
- Maclaurin series and the general Taylor series centered at $x = a$.
- Maclaurin series for the functions e^x , $\sin x$, $\cos x$, and $\frac{1}{1-x}$

- Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation and the formation of new series from known series.
- Functions defined by power series.
- Radius and interval of convergence of power series.
- Lagrange error bound for Taylor polynomials.

Evaluation:

Marking Period grades will be determined as follows:

Performance assessments	90%
Homework	10%

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